MINDARIE SENIOR COLLEGE

MATHEMATICS SPECIALIST 3CD

SEMESTER 1 2010

TEST 2

	Questions	Reading Time	Working Time	Marks
Calculator Free	**1 – 4**	**5 minutes**	**15 minutes**	**12**
Calculator Assumed	5 - 8	5 minutes	30 minutes	24
Total				36

1. [1, 2 marks]

Express in exact ~~polar~~ rectangular form:

(a) $e^{i\frac{\pi}{6}}$

$= \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2} + i\frac{1}{2}$ ✓

(b) $3e^{2+i\frac{2\pi}{3}}$

$= 3e^2 e^{i\frac{2\pi}{3}}$ ✓

$= 3e^2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$= 3e^2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$ ✓

$= -\frac{3}{2}e^2 + i\frac{3\sqrt{3}}{2}e^2$

2. [2 marks]

Given that $z = 3e^{i\theta}$, determine an expression in exponential form for iz.

$iz = e^{i\frac{\pi}{2}} \cdot 3e^{i\theta}$ ✓

$= 3e^{i(\theta+\frac{\pi}{2})}$ ✓

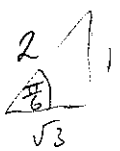
3. [2, 1, 1 marks]

Given that $w = \sqrt{3} + i$, express in exact exponential form:

(a) $w = 2e^{i\frac{\pi}{6}}$ ✓ ✓

(b) $\overline{w} = 2e^{-i\frac{\pi}{6}}$ ✓ ✓

(c) $w^3 = 8e^{i\frac{\pi}{2}}$ ✓

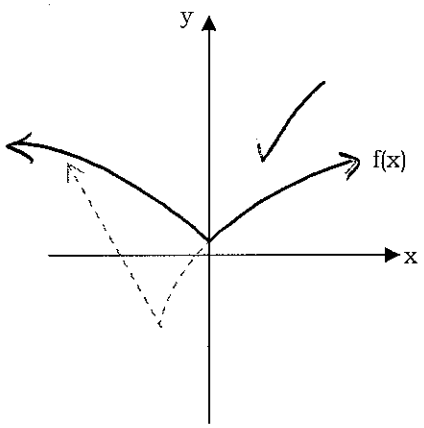


4. [1, 2 marks]

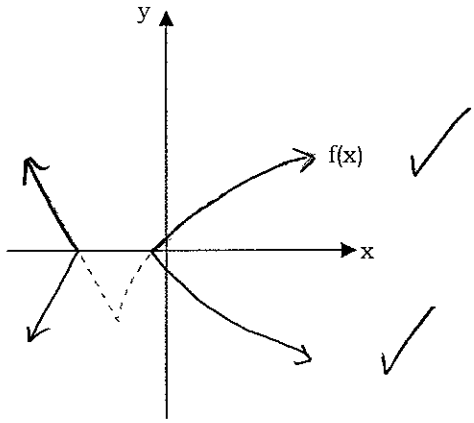
The sketch of y = f(x) is given below.

Sketch on the same axes the graphs of:

(a) $y = f(|x|)$



(b) $|y| = f(x)$



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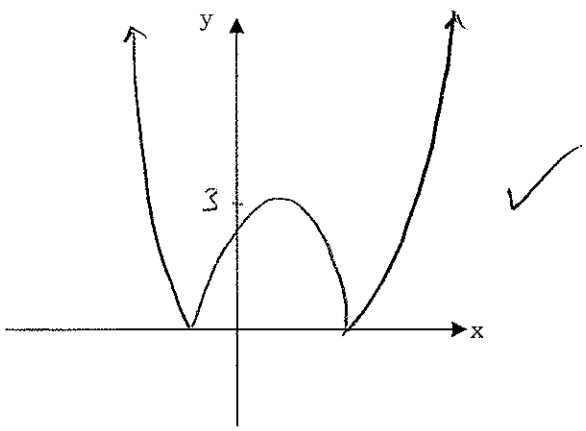
5. [2 marks]

Solve for x: $\sqrt[3]{3+|x|} \leq 2$

$-5 \leq x \leq 5$ ✓✓

6. [1, 1, 2 marks]

(a) Sketch the graph of $f(x) = |x^2 - 2x - 2|$.



✓

Hence state the value(s) of b such that f(x) = b has exactly

(a) three solutions $b = 3$ ✓

(b) two solutions $b > 3$ ✓ or $b = 0$ ✓

7. [1, 4, 5 marks]

Consider the plane Π_1: $\mathbf{r} = (3 + 2\mu - \lambda)\mathbf{i} + (5 - 4\mu + 2\lambda)\mathbf{j} + (7 + 3\mu + 3\lambda)\mathbf{k}$.

(a) Find the equation of the plane containing the point (2, 1, 5) and parallel to Π_1.

$$\underset{\sim}{r} = (2 + 2\mu - \lambda)\underset{\sim}{i} + (1 - 4\mu + 2\lambda)\underset{\sim}{j} + (5 + 3\mu + 3\lambda)\underset{\sim}{k}$$ ✓

(b) Determine if the point (13, -9, 4) lies on the plane Π_1.

$13 = 3 + 2\mu - \lambda \Rightarrow 10 = 2\mu - \lambda$ — (1)

$-9 = 5 - 4\mu + 2\lambda \Rightarrow -14 = -4\mu + 2\lambda$ — (2) ✓

$4 = 7 + 3\mu + 3\lambda \Rightarrow -3 = 3\mu + 3\lambda$ — (3)

using (2) & (3)

$\mu = 2 \quad \lambda = -3$ ✓

$10 = 2(2) - (-3)$

$= 4 + 3$

$= 7$ ✓ No. ✓

✓ Equating
✓ solving
✓ subst.
✓ Answer

(c) Write Π_1 in normal form (ie $\mathbf{r}\cdot\mathbf{n} = \mathbf{c}$).

Need vector $\perp$ to $2i - 4j + 3k$ and $-i + 2j + 3k$

$(a\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k}) \cdot (2\underset{\sim}{i} - 4\underset{\sim}{j} + 3\underset{\sim}{k}) = 0$ ✓

$2a - 4b + 3c = 0$ — (4)

$(a\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k}) \cdot (-\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}) = 0$ ✓

$-a + 2b + 3c = 0$ — (5)

(4) − (5)

$3a - 6b = 0$

$a = 2b$

So $a = 2 \quad b = 1 \quad c = 0$ ✓

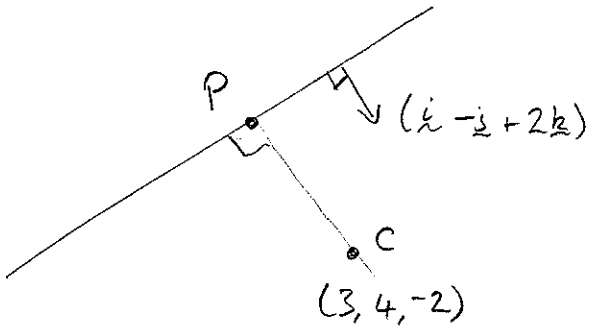
$\underset{\sim}{r} \cdot (2\underset{\sim}{i} + \underset{\sim}{j}) = (3\underset{\sim}{i} + 5\underset{\sim}{j} + 7\underset{\sim}{k}) \cdot (2\underset{\sim}{i} + \underset{\sim}{j})$ ✓

$= 6 + 5$

$\underset{\sim}{r} \cdot (2\underset{\sim}{i} + \underset{\sim}{j}) = 11$ ✓

8. [8 marks]

A sphere centred at (3, 4, -2) and with radius 2 can be defined by the vector equation $|\mathbf{r} - (3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}| = 2$. Determine the minimum distance the sphere is from the plane defined by $x - y + 2z = 13$.



Line CP: $\underset{\sim}{r} = (3\underset{\sim}{i} + 4\underset{\sim}{j} - 2\underset{\sim}{k}) + \lambda(\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k})$

$= (3+\lambda)\underset{\sim}{i} + (4-\lambda)\underset{\sim}{j} + (-2+2\lambda)\underset{\sim}{k}$ ✓✓

Intersection of line & plane.

Plane: $\underset{\sim}{r} \cdot (\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}) = 13$ ✓

$\therefore [(3+\lambda)\underset{\sim}{i} + (4-\lambda)\underset{\sim}{j} + (-2+2\lambda)\underset{\sim}{k}] \cdot (\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}) = 13$ ✓

$3 + \lambda - 4 + \lambda - 4 + 4\lambda = 13$

$6\lambda = 18$

$\lambda = 3$ ✓

$|(\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k})| = \sqrt{1+1+4}$

$= \sqrt{6}$ ✓

Dist C to P $= 3\sqrt{6}$ ✓

$\therefore$ Dist Plane to sphere $= 3\sqrt{6} - 2$ ✓